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"SWEET SPOT" OR "SWEET ZONE"? MODAL ANALYSIS OF A WOODEN BASEBALL BAT FOR DESIGN OPTIMIZATION

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ABSTRACT

This paper describes modal testing of a wooden baseball bat with the purpose of finding the peak frequencies and vibration modes and their relation to the so-called "sweet spot". Initial vibration testing was done by performing a tap test along a Louisville Slugger wooden baseball bat. The bat was suspended by elastic rubber bands to approximate free-free boundary conditions. The peak frequencies and bending mode shapes of the baseball bat were obtained. To verify the results, a modal analysis of the baseball bat was performed to simulate the structure's dynamic behavior. The animated model validated the accuracy of the parameters obtained in the tap test. The bending mode shapes were also compared to the bending elastic mode shapes of a uniform, homogeneous beam undergoing no shear distortion. The exact solution of the beam equation of motion was solved. The mode shapes were plotted to compare them with the ones obtained from the baseball bat. This comparison indicated that the bat and the beam structures presented the same type of bending mode.

I. INTRODUCTION

Modal analysis represents a reliable and important technique to study a structure's dynamic characteristics, including its natural vibration frequencies and mode shapes. The intention of conducting this project was to carry out a modal analysis of a wooden baseball bat as part of a larger effort to find the principal modal parameters of the bat structure, such as the center of percussion (COP), the peak frequencies, main nodes, and the vibrational mode shapes along the bat as well as their relation to the so-called "sweet spot", which will be shown to be more of a "sweet zone".

Two vibration measurement techniques were used to determine the modal parameters of the baseball bat. The initial testing was done by carrying out a tap test along a Louisville Slugger wooden baseball bat. The bat was suspended by elastic bands to approximate free-free boundary conditions. The second test was a full modal analysis using STARModal® software. The animated modes validated the accuracy and reliability of the modal parameters obtained in the initial tap test. In addition, a uniform, homogeneous beam undergoing no shear distortion was used to compare the bending mode shapes of the baseball bat with the mode shapes of the beam.

The results of these analysis techniques were used as a baseline to propose a new and more enhanced baseball bat design, which will take into consideration the dynamic and vibrational behavior of the baseball bat structure.

II. EXPERIMENTATION AND ANALYSIS

A baseball bat can be viewed as an ordinary structure used to play a game. However, there is nothing simple that can define a baseball bat. A wood baseball bat is a homogeneous orthotropic, elastic bar with changing cross section. Its shape produces a very complex structure, which affects the dynamic behavior and produces significant properties that govern the bat [6]. Therefore, the best way to understand the behavior of a baseball bat is to use a modal analysis approach, which helps define the parameters of a structure for all the elastic modes in the frequency range of interest. The modal frequency and mode shapes parameters form a complete description of the inherent dynamic characteristics of the baseball bat [3].

Using modal analysis, a deflection pattern of the baseball bat is resolved into a set of simple mode shapes with individual peak frequencies. A mode shape is a deflection pattern associated with a particular modal frequency, which defines the deflection pattern as if that mode existed in isolation from all other modal frequencies in the structure [3].

A modal analysis of a baseball bat can help identify the important locations along of the bat since it will help establish the positions and vibrational properties of the center of percussion, peak frequencies, and the node locations of the vibration modes.

The center of percussion as well as the first and second mode of vibration conform the sweet *"zone"* of the baseball bat are shown in Fig. 1. The sweet zone is the area on the bat barrel where *"the transfer of energy from the bat to the ball is maximal while the transfer of energy to the hands is minimal"* [4]. That is, the ball will travel the farthest and with the greatest output velocity.

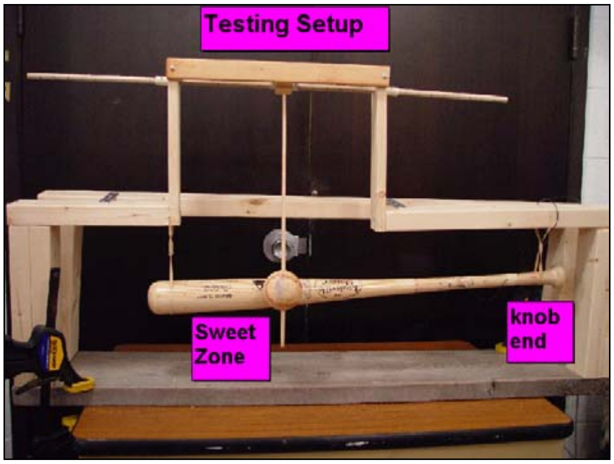

Figure 1. Testing Setup for "Sweet Zone"

The center of percussion (COP) is the point located at the barrel of the baseball bat where the collision between the ball and the bat does not produce any reaction impulse at the axis; that is, preventing any shock on the hand of the batter [4].

In addition, the bending modes of vibration represent the points of maximum natural deflection in response to the ball's collision [5]. The first and second nodes located at the barrel end produced by these bending modes of vibration represent the optimal locations to hit the bat since it will not cause the natural frequencies corresponding to these nodes to become "excited"; therefore, there will not be any energy loss due to these frequencies vibrating along the bat. The position of the sweet zone on the baseball bat barrel can be depicted in Fig. 2.

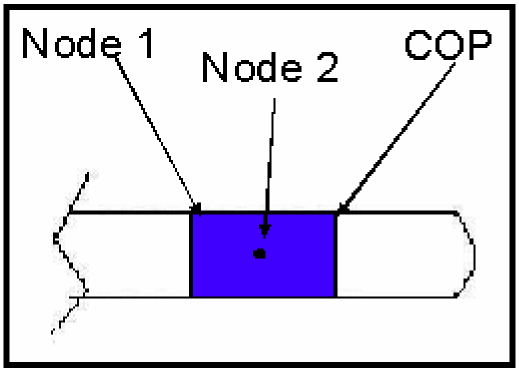

Fig. 2. Distinct Components of the "Sweet Zone"

Another relevant characteristic is the determination of the reaction forces produced at the moment of the ball impact, which are shown in Fig. 3. The reaction forces may be determined by calculating the average acceleration at each location point along the bat.

Fig. 3. Testing for Reaction Force at Impact Velocity

III. PROCEDURE

Modal Testing was performed on a Louisville Slugger wooden baseball bat under free-free boundary conditions. There were two tap vibration tests done on the baseball bat: the Tap Test and the COP Test. For both tests, a Brüel & Kjaer type 8202 modal hammer, displayed in Fig. 4, was utilized, that will cause the excitation on the bat. A rubber tip was used on the impact hammer since it excited the lower modes of vibration of interest for this particular modal test.

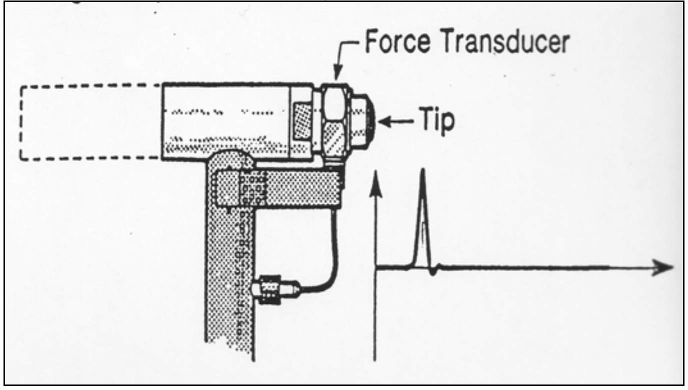

Fig. 4. Type 8202 Modal Hammer [3]

The impact results were stored and displayed on a type 2032 dual-channel FFT Analyzer manufactured by Brüel & Kjaer [3].

To obtain the feedback from the impact with the hammer, an accelerometer was placed on assigned locations along the baseball bat. The accelerometer used was a type 4374-miniature accelerometer also manufactured by Brüel & Kjaer (see Fig. 5). The accelerometer was set and calibrated 1g using a vibration calibrator.

To start the modal test, the frequency range was adjusted to determine the bandwidth and resolution. The trigger was determined by tapping several times until the analyzer auto-ranged the data at that particular force. A curve fitting of the data was performed to fit all the measured data in an automatic sequence on the analyzer.

For the Tap Test, 81 nodes were created and tapped along the bat. The points at which the relative amplitudes were zero indicated that these were node locations. This method helps identify the first and second bending modes of vibration.

Fig. 5. Bruel & Kjaer 4374 Accelerometer

For the COP Test, 17 sets of circumferential points were drawn. There were six points in each circle near the barrel and five on the handle. A total of 12 circumferential points near the barrel and the remaining 5 near the handle were created for a total of 145 points. The baseball bat was tapped three times on each node along the bat while monitoring coherence with the purpose of utilizing this data to verify the node locations using a software simulation.

At the same time, acceleration locations were considered to simulate the batter's hand at locations A, B, C, D (Table 3) where the accelerometer was placed at these locations. A ball fixed to a pendulum was dropped from a known height on several locations along the barrel and the reaction forces at positions A, B, C, and D were recorded to determine the reaction forces at these locations.

To validate the vibration tap tests, an animated model was created to visualize the mode shapes along the baseball bat. All the nodes were measured and transformed into polar coordinates (r, θ, z) for inclusion to the STARModal® software, version 5.24.32. The accelerometer was fixed at the different locations A, B, C and D and frequency response function measurements were recorded for each location on the STARModal® software. These measurements were plotted and an animated baseball bat model was created for the first and second bending mode shapes.

The exact solution of the beam equation of motion was solved using separation of variables. To solve the equation, the beam was assumed to be uniform, homogeneous, and with no shear force acting on it. The general solution of the beam equation has the form [2]:

$$EI\frac{\partial^4 y}{\partial x^4}+\gamma\frac{\partial^2 y}{\partial t^2}=0 \qquad \text{(Eq. 1)}$$

E= Modulus of elasticity
I=Area moment of inertia
γ=mass per unit length

The solutions for the roots of the exact solution were obtained and the modes of vibration for the beam are plotted to evaluate the mode shapes with the ones obtained for the baseball bat.

IV. RESULTS

A Tap Test facilitates the determination of the peak frequencies for the first and second modes of vibration as shown in Fig. 6. The first bending mode of vibration is represented in Fig. 7. For the second bending mode, two different peak frequencies were obtained to represent the same mode of vibration. These are actually the same mode shape offset by 90° from each other. These mode shapes are depicted on Fig. 8 and 9.

For the Center of Percussion (COP) Impact Test, the results obtained are summarized on Fig. 10, which shows the reaction forces obtained along locations A, B, C, and D on the baseball bat.

A tabulated summary is also displayed from the testing and experimentation results. The natural frequencies and specific identification of their respective locations along the baseball bat are shown in Table 1. The acceleration locations as well as the reaction forces obtained for each specific location are displayed on Table 2 with two significant figures of accuracy. The Center of Percussion (COP) positions according to locations A to D are listed in Table 3 with three significant figures of accuracy.

The exact solution of the beam equation is resolved into the following equation:

$$W_r(x) = [(\cos\beta_r L - \cosh\beta_r L)(\sin\beta_r x + \sinh\beta_r x) - (\sin\beta_r L - \sinh\beta_r L)(\cos\beta_r x + \cosh\beta_r x)] \quad \text{(Eq. 2)}$$

where *r* = *2, 3, …*

W_r = Eigenfunctions belonging to $\beta_r L$
$\beta_r L$= Roots of the beam equation
L= Length of the beam, same bat length (m)
x= Length increments (m)

Using MAPLE® software, the roots of the exact solution were identified by plotting Eq. 2. These solutions are plot in Fig. 13. The eigenfunction equation for the first and second mode shape was graphed in order to compare these mode shapes with the modes of vibration obtained for the baseball bats. These plots are shown in Fig. 14 and 15.

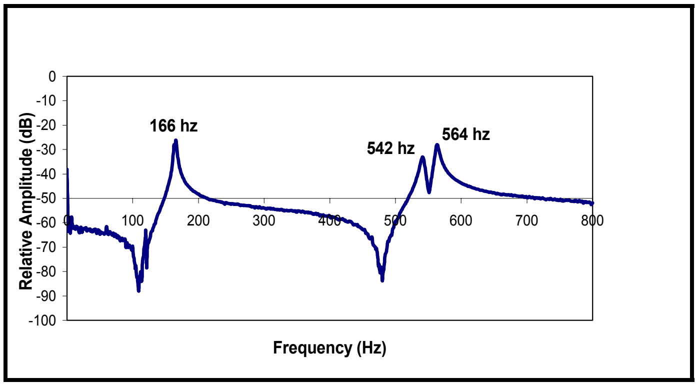

Fig. 6. Driving Point Locations

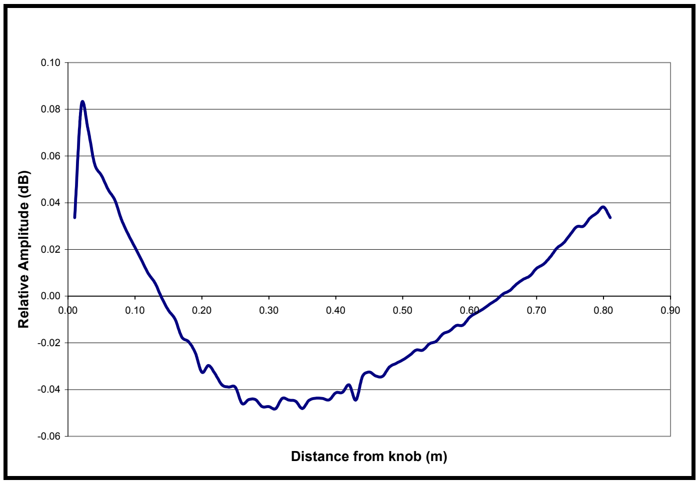

Fig. 7. Node Locations at 166 Hz (First Mode)

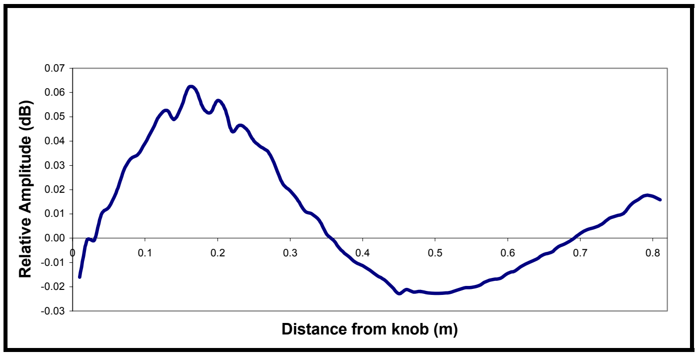

Fig. 8. Node Locations at 542 Hz (Second Mode)

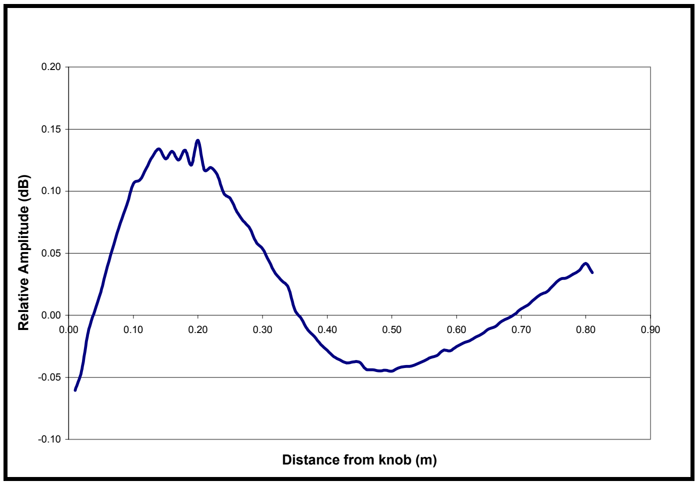

Fig. 9. Node Locations at 564 Hz (Second Mode)

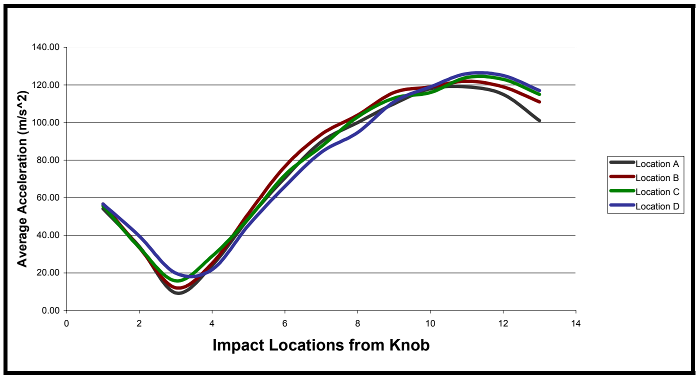

Fig. 10. Impact Locations

Table 1. Frequencies and Node Locations

Mode	Frequency (Hz)	Node Location along axis starting at the knob (m)
1	**162.4**	**0.2, 0.6**
1	**165.6**	**0.2, 0.6**
2	**536.99**	**0.05, 0.4, 0.7**
2	**559.84**	**0.05, 0.4, 0.7**

Table 2. Acceleration Locations and Reaction Forces

Location of Acceleration	Average Acceleration (m/s²)/ Reaction Force (N)
Location A	**9.4/1.3**
Location B	**12./1.6**
Location C	**16./2.1**
Location D	**20./2.7**

COP Test Location	Distance from Knob (m)
Point A	**0.708**
Point B	**0.682**
Point C	**0.678**
Point D	**0.678**

Table 3. Center of Percussion Location

Figures 11 and 12 are animated versions of the baseball bat using the modeling software to verify results obtained experimentally. These figures show the mode shapes as well as the peak frequencies of the first and second mode shape, respectively.

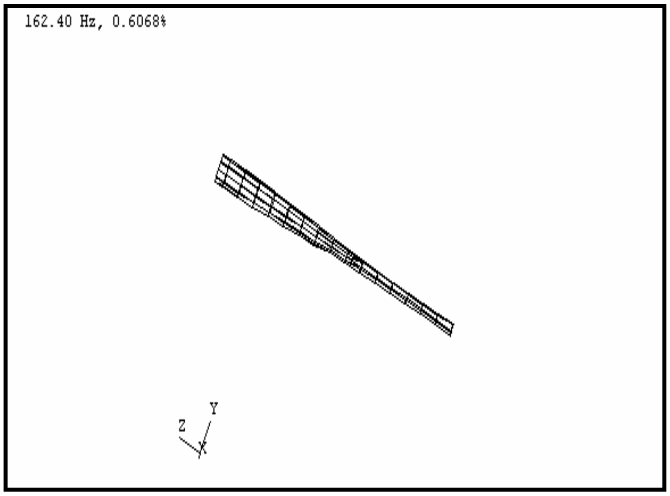

Fig. 11. Animation of First Mode Shape (162.40 Hz)

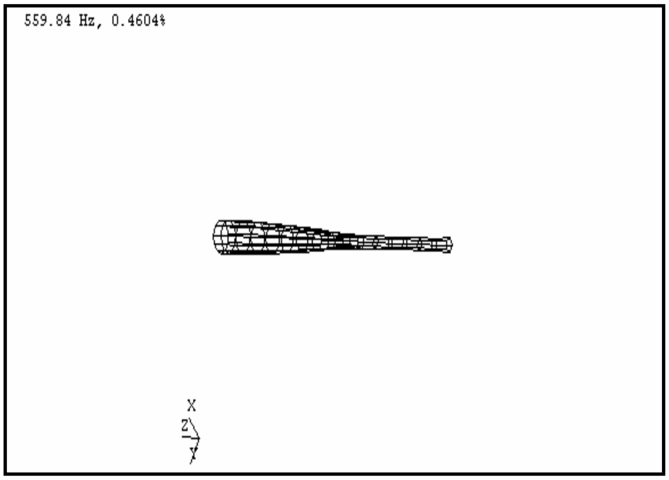

Fig. 12. Animation Second Mode Shape (559.84 Hz)

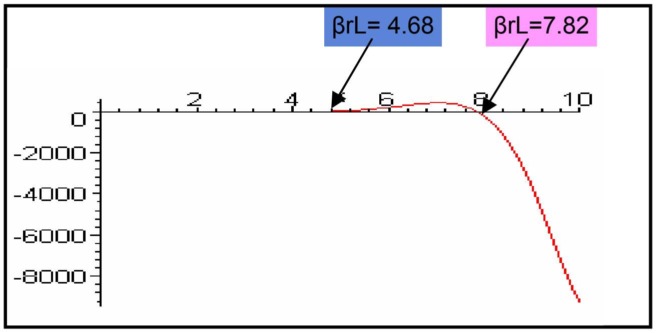

Figure 13. Maple Plot of Roots for Beam Equation

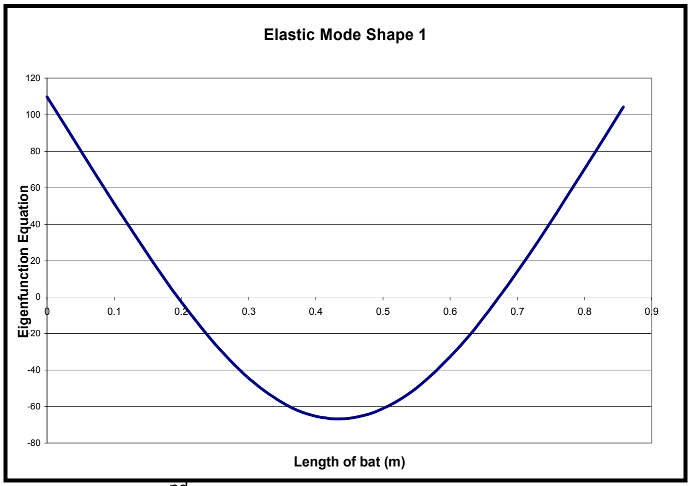

Figure 14. 2nd Elastic Mode Shape of Beam Equation

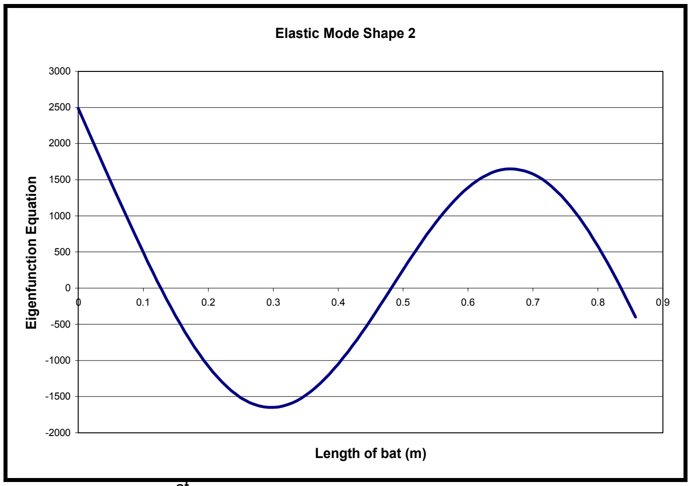

Figure 15. 1st Elastic Mode Shape of Beam Equation

V. CONCLUSIONS

The modal analysis of the wooden baseball bat provided a comprehensive and detailed study of the principal locations of the bat structure, such as the center of percussion (COP), the peak frequencies, main nodes and the two vibrational bending mode shapes. Although the results differed slightly with the published literature, the data obtained is compiled on the same range of numbers.

There were many uncertainties that could have possibly affected the data. First, the bat-ball contact area was very vague since the area of contact was somehow significant and varies at every impact. Second, the node point locations along the bat were placed as accurate as possible; however, these points were man-made point locations, which continuously increase the error in the calculation. There was a tendency of error when drawing the nodes by hand. Third, the diameter of the modal hammer contributes to uncertainties and errors since it was significantly high. The hammer’s diameter was 0.016 m, which causes a relatively large area of contact when it tapped the node on the baseball bat. And fourth, the isotropic material assumption; that is, the properties exhibit on the bat materials were assumed to be the same along all the directions. Therefore, this assumption did not consider the wood grain direction, which alters the configuration and properties of the bat.

The key factor found in this project was that the “sweet spot” is actually a "sweet zone" composed not only of the center of percussion but also of the two nodes. These three important locations are so close to one another that it was difficult to differentiate. The zone is characterized by having the lowest average acceleration on the baseball bat and therefore the lowest reaction force, which means that all the impact force was taken by the ball providing a high post-impact velocity.

The peak frequencies were also an important finding in this analysis since these frequencies displayed the first and second mode along the bat. The peak frequency, 166 Hz, represented the first mode, and frequencies 542 and 564 Hz were the same second mode of vibration at a different orientation. This was corroborated with the modal analysis software, which showed the vibrational mode at the different frequencies.

The modal analysis that was performed also determined that the average acceleration at different locations did not differ along the locations as it was expected. This occurred because the average acceleration and the reaction forces obtained were with respect to the knob-end location. That is, the location where the bat was suspended by elastic the rubber bands. And at first it was expected that the average acceleration be obtained with respect to the specified locations A, B, C or D.

Although the testing evaluated and analyzed the different locations along the bat, these findings did not grasp in a real manner the reality of playing the game of baseball. There are many other factors that must be taken into consideration for future research when swinging a bat and hitting a ball. These factors are more related to human physiology and biomechanics of the individual batter. Factors such as grip firmness affect the pattern and performance when swinging a baseball bat. There is a whole muscle pattern and dynamics when this action is performed, which changes from individual to individual.

Nevertheless, the modal analysis and testing embodies an important approach to identify the different modes of vibration along the baseball bat. At the same time, these findings serve as a baseline for designing an optimal training baseball bat that will help the batter develop his training skills by enhancing the "sweet zone".

The similarity of the baseball bat with the beam was another key factor to understand the behavior of the baseball bat when it undergoes vibrational excitation. Finding the mode shapes of the baseball bat and the beam using distinct approaches help understand the critical factors that governed the characteristics of the baseball bat. This makes it easier to control the different aspects of the baseball bat and modify its significant areas.

Consequently, a great improvement has been achieved in understanding the behavior of such a simple structure as a baseball bat. Most of the initial objectives were met and developed into more complex objectives that require a detail analysis regarding this structure.

VI. REFERENCES

[1] Baruh, H., 1999, "Analytical Dynamics". McGraw-Hill. New York. pp. 26-32.

[2] Benaroya, H., 1998, "Mechanical Vibration: Analysis, Uncertainties, and Control". Prentice Hall. New Jersey. pp. 413-432.

[3] Bruel & Kjaer. Structural Testing: Part 2- Modal Analysis and Simulation. Bruel & Kjaer. Denmark. pp. 4-48.

[4] Noble, L. Inertial and Vibrational Characteristics of Softball and Baseball Bats: Research and Design Implications. http://www.isbs98.uni-konstanz.de/fullpaper/lnoble

[5] Russell, D. A. 2000. Acoustics of Baseball Bats: Vibrational Bending Modes and the Sweet Spot. Science and Mathematics Department. Kettering University.

[6] Zandt, L.L., 1992, "The Dynamical Theory of the Baseball Bat". American Journal of Physics. 60 (2), pp.172-181.